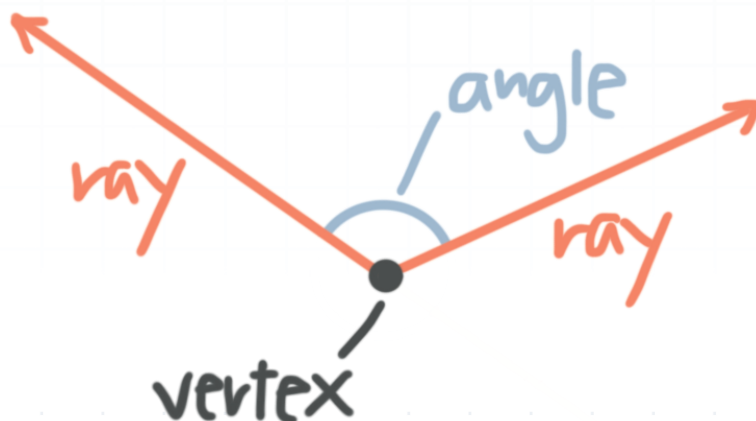


Complementary and supplementary angles

Introduction to angles

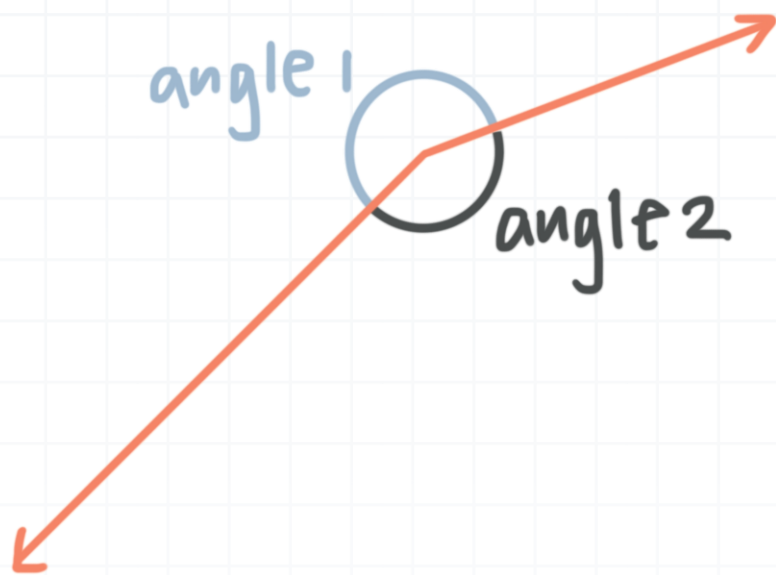
You can think of an **angle** as a wedge, like a piece of pie. Each side of an angle is bounded by a **ray**, which is a line that's infinitely long in only one direction. So a ray has an endpoint on one end, and goes off infinitely on the other end.

You get an angle when you put two rays together, with their endpoints at the same spot. That becomes the corner of the angle, which is called the **vertex** of the angle.



Every angle has its interior area, but there's also the area that's outside the angle. When you cut a pie, you have the area of the piece you cut, but you also still have the area of the rest of the pie.



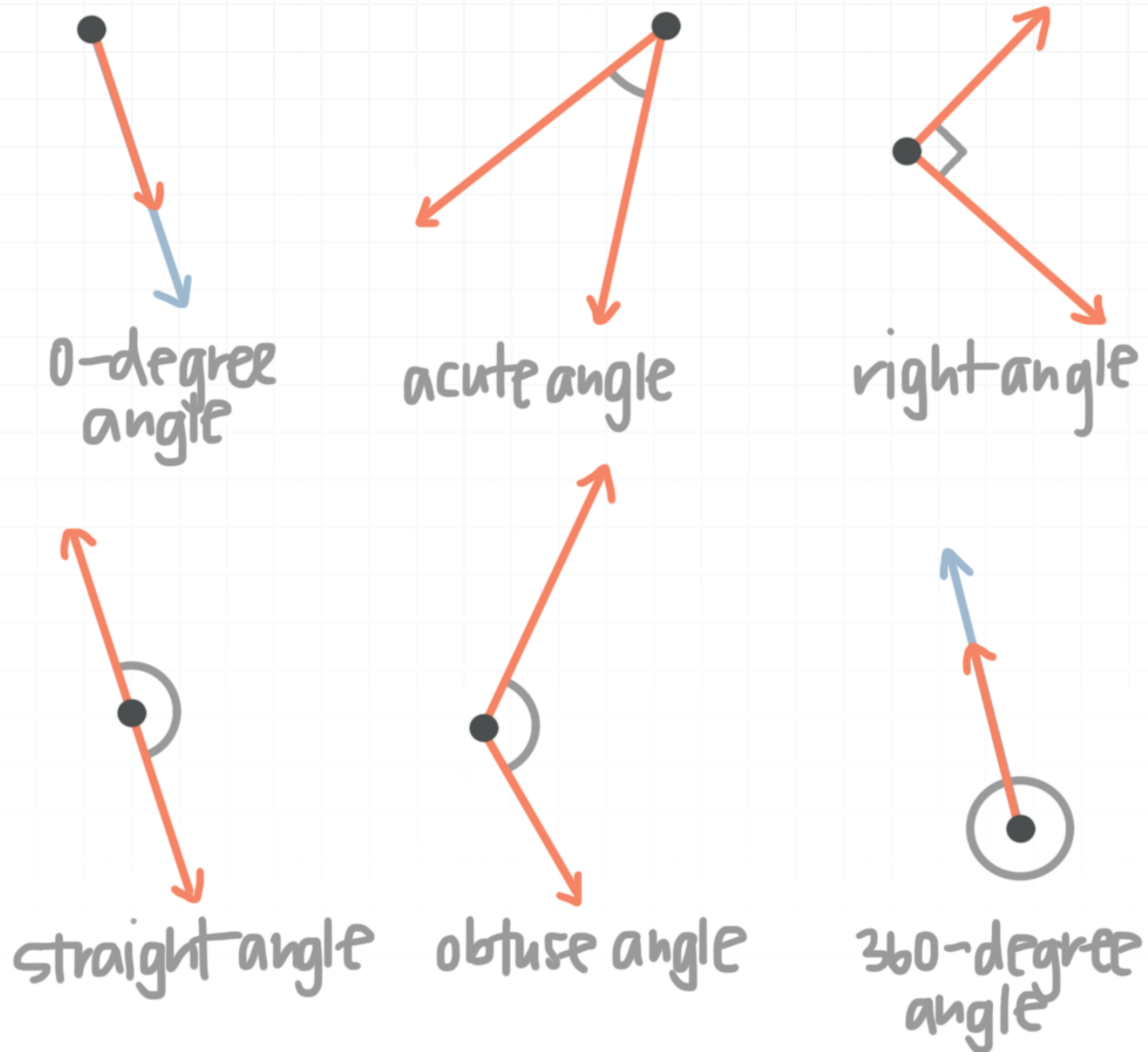


We usually use θ for the measure of an angle, but you can also use α , β , γ , or any other variable. If you cut out $1/4$ of a pie, you'd be cutting out a quarter of a 360° pie, so the angle measure of the piece you cut would be $360^\circ/4 = 90^\circ$. Here are some facts we want to know about angles.

1. A full circle is always $\theta = 360^\circ$.
2. When the sides of the angle overlap (there's no area in between the rays), then the angle measure is $\theta = 0^\circ$.
3. When you take exactly one quarter of the circle, the sides of the angle are perpendicular, and the angle is $\theta = 90^\circ$. 90° angles are called **right angles**.
4. When you take exactly half of the circle, the sides of the angle form a straight line, and the angle is $\theta = 180^\circ$. 180° angles are called **straight angles**.
5. When $0^\circ < \theta < 90^\circ$, we call it an **acute angle**.
6. When $90^\circ < \theta < 180^\circ$, we call it an **obtuse angle**.

Here's what these angles look like.





We can also express angles in radians instead of degrees. As we go further in math, we'll actually use radians more often than degrees. We write radian angles as " $\theta = 0.456$ radians", " $\theta = 0.456$ rad", or just as $\theta = 0.456$.

It's really common to see radian angles given in terms of π , like $\pi/3$, π , or 1.6π . Here are some important angles in radian measure.

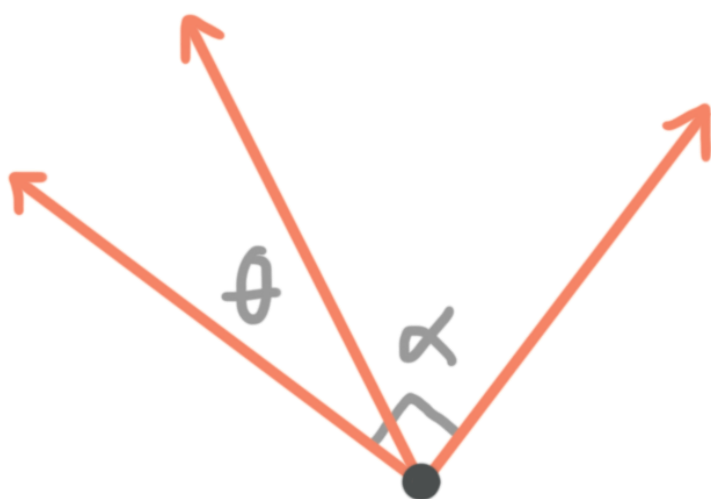
1. A full circle is always $\theta = 2\pi$.



2. When the sides of the angle overlap (there's no area in between the rays), then the angle measure is $\theta = 0$.
3. When you take exactly one quarter of the circle, the sides of the angle are perpendicular, and the angle is $\theta = \pi/2$.
4. When you take exactly half of the circle, the sides of the angle form a straight line, and the angle is $\theta = \pi$.
5. Acute angles are angles with $0 < \theta < \pi/2$.
6. Obtuse angles are angles with $\pi/2 < \theta < \pi$.

Complementary angles

Complementary angles are two angles that sum to 90° or $\pi/2$ radians. In the following figure, θ and α are complementary because when you put them together they form a right angle.



Example

Find the angle θ that's complementary to 37° .



The angle that's complementary to 37° is whatever angle we have to add to 37° in order to get 90° . Therefore,

$$37^\circ + \theta = 90^\circ$$

$$\theta = 90^\circ - 37^\circ$$

$$\theta = 53^\circ$$

We can find complementary angles in radians, too. For example, the angle that's complementary to $\pi/6$ is

$$\frac{\pi}{6} + \theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - \frac{\pi}{6}$$

$$\theta = \frac{6\pi}{12} - \frac{2\pi}{12}$$

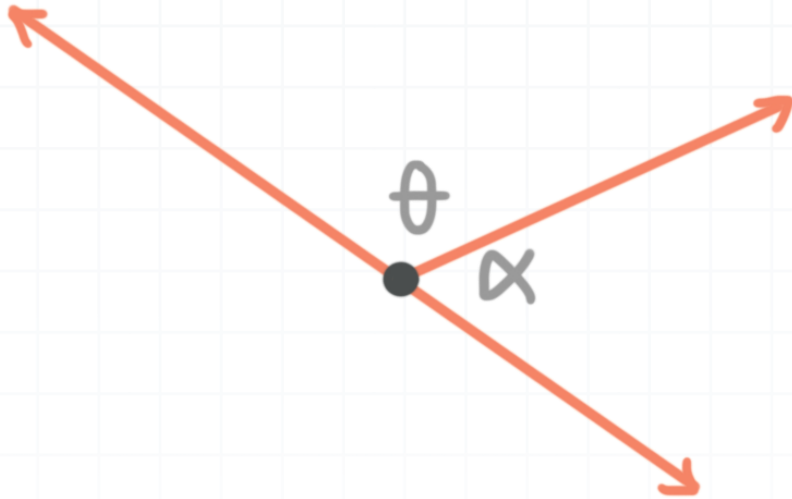
$$\theta = \frac{4\pi}{12}$$

$$\theta = \frac{\pi}{3}$$

Supplementary angles



Supplementary angles are two angles that sum to 180° or π radians. In the picture below, θ and α are supplementary because when you put them together they form a straight angle.



Example

Find the angle θ that's supplementary to $\pi/4$.

The angle that's supplementary to $\pi/4$ is whatever angle we have to add to $\pi/4$ in order to get π . Therefore,

$$\frac{\pi}{4} + \theta = \pi$$

$$\theta = \pi - \frac{\pi}{4}$$

$$\theta = \frac{4\pi}{4} - \frac{\pi}{4}$$

$$\theta = \frac{3}{4}\pi$$



We can find supplementary angles in degrees, too. For example, the angle that's supplementary to 48° is

$$48^\circ + \theta = 180^\circ$$

$$\theta = 180^\circ - 48^\circ$$

$$\theta = 132^\circ$$

Let's do a slightly more complicated example.

Example

Find the angle θ that's twice the supplement to $\pi/3$.

If two angles are supplementary, they sum to 180° or π radians. The angle that's supplementary to $\pi/3$, which we'll call α , is therefore

$$\alpha = \pi - \frac{\pi}{3}$$

$$\alpha = \frac{3\pi}{3} - \frac{\pi}{3}$$

$$\alpha = \frac{2}{3}\pi$$

We were asked to find the angle that's twice as big as α , so we'll multiply by 2.



$$\theta = 2\alpha$$

$$\theta = 2\left(\frac{2}{3}\pi\right)$$

$$\theta = \frac{4\pi}{3}$$

